Number Line Device
GUIDEBOOK

American Printing House
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A revision of *Guide to the APH Number Line Device*
by Eleanor Pester produced by APH.

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NUMBER LINE DEVICE

The number line is a basic tool for teaching number concepts, number sequence, counting, addition, subtraction, multiplication, division, rounding, estimating whole numbers, common fractions, and decimals.

The Principles and Standards for School Mathematics states that students K-12 should "Understand numbers, ways of representing numbers, relationships among numbers, and number systems" (National Council of Teachers of Mathematics, 2000, p. 32). Number Lines can be a useful tool in meeting the standards as students determine the relative magnitude of whole numbers, the effects of addition and subtraction of whole numbers, compare and order fractions, explore numbers less than zero, and graph algebraic equations.

The Common Core State Standard for Mathematics (2010) directly references the use of number lines to teach mathematical concepts. The standards range from representing whole number sums on a number line at the elementary level, to representing fractions on a number line at the intermediate level, and locating irrational numbers on a number line at the middle school level.

Number lines are a valuable tool for students with visual impairments. Kapperman (1997) notes:

> The Number Line can be used for working on operations, relationships, fractions, and decimals. Number lines are especially useful when they are stretched across the top of the student's desk. Students can find the larger number, count forward for addition and count backward for subtraction. Students can use number lines for working on positive and negative numbers as well. (p. 23)

The Number Line Device is designed for use by students who use large print and braille. The device can be used to supplement any math program and to support math concepts taught at any level.

Use the number line device and line segments with pegs and/or bars to help the student develop the following concepts about number lines intuitively:

1. Number lines usually have equally spaced marks with numerals under each mark (beside each mark if the number line is presented vertically).

2. The zero mark is usually the beginning mark but may be positioned at any convenient place on the line.

3. The numeral under each mark tells how far it is from the beginning mark (beside each mark if the number line is presented vertically).

4. The numbers named along a number line become greater from left to right and bottom to top.

5. The number line goes on and on (infinitely) at both ends and is represented by only a small segment.

6. The number line is usually horizontal, but can also be vertical or even round.

7. Some commonly used number line segments are rulers, yardsticks, thermometers, and clocks.

DESCRIPTION OF THE DEVICE

The number line device consists of three parts: a blue plastic base, fifteen interchangeable vinyl line segments, and forty plastic cylinders, which serve as pegs and/or interlocking bars. There are ten additional plastic cylinders/pegs with an arrow formation on one end that may be used by older math students as they graph lines and rays.

The plastic base houses one or two of the vinyl line segments at a time. The segments are fastened securely to the base over a peg located at each end of the base. On the base behind each line segment is a horizontal groove with holes at intervals. This horizontal groove corresponds to the line on the segment while the holes represent the coordinate scale marks.

The plastic base also contains two work/storage areas. The large work/storage area at the bottom of the base can be used to store the number strips or as a work space during instruction. The smaller work/storage area on the right of the base can serve as a storage area for the pegs or as a work space during instruction.

Each vinyl line segment is white with a black raised line running the length of the strip. Large raised black dots are located at intervals on the raised line and indicate the position of the braille and print numerals on the segment. (Braille numeric indicators have been eliminated on all segments to facilitate reading and placement of the numbers on the strip). The following line segments marked horizontally are included:

1. 0–20 by ones
2. 0–200 by tens
3. 0–100 by fives
4. -10 – +10 by ones
5. Halves
6. Thirds
7. Fourths
8. Fifths
9. Sixths
10. Eighths
11. Fractional tenths
12. Decimal tenths
13. Unnumbered with 10 coordinate scale marks
14. Unnumbered with 20 coordinate scale marks
15. Unnumbered with no coordinate scale marks

The unnumbered line segments have been included in the kit to provide the teacher flexibility in designing problems. When using the unnumbered line segments use APH Feel 'n Peel Nemeth Braille/Print Number Stickers to mark the number line with the appropriate numbers. You may use graphic art tape to make the coordinate scale marks on the "blank" number lines.

Some suggestions for using the unnumbered lines are as follows:

1. 0–1000 by hundreds

2. Skip counting by two, three, four, or any number

3. 20–40 by ones horizontally to extend the beginning number line segment concepts as needed

4. -60° – +130° by tens vertically to simulate the markings on a common mercury thermometer

5. Graphing simple algebraic expressions

The forty white plastic cylinders are used to show numerical relationships on the number line segments. They are hollow on one end with a corresponding projected tip on the other end. The tip and opening enable the cylinders to be interlocked. When they are interlocked and placed horizontally in the base groove, the cylinders are called bars. Bars represent the segments on the number lines. When the cylinders are placed individually and vertically in the base holes, they

are called pegs. The pegs represent the coordinate scale marks on the number lines.

Ten pegs have been included that have "arrows" at the tip to represent rays or lines. Older students may find these useful when graphing inequalities.

WORKING WITH THE NUMBER LINE

1. Place the base on a table or desk directly in front of the student. When using horizontally marked line segments, the large work/storage area for the line segments on the base should be nearest to and parallel to the student's body. When using vertically marked line segments, the base should be perpendicular to the student's body with the large work/storage area to the right if the student is right-handed and to the left if the student is left-handed.

2. Place the desired line segment(s) over the end pegs protruding from the plastic base. A hole at each end of each segment aligns with the pegs in the base to hold the segment securely. When using only one segment on the device, place it on the lower level nearest the large work/storage area for convenience.

3. Put a suitable number of pegs/bars (a few more than will be required for the problems being done) in the smaller work/storage area.

4. Using a problem given by the teacher or one taken from the mathematics textbook, count the designated number of pegs/bars for the first part of the problem into the work/storage area.

5. Decide whether to use pegs or bars to work the problem. Pegs emphasize points and counting, but beware of "skipping errors" – errors caused by skipping holes in the base. Bars emphasize line segments and lengths. When working with fractions, use bars to show parts of one whole. Use pegs to show parts of a set. If desired, use both pegs and bars to show contrast. For example, use pegs to illustrate ones and bars to illustrate tens.

6. After counting the pegs/bars into the work area, next place them in the groove or holes on the base corresponding to the line segment or point desired. The number to the right or above zero is usually used as the starting point.

   When using pegs, place the tip of the peg in the groove and slide it to the right for a horizontally marked segment, or place the peg above the last peg inserted for a vertically marked segment. Slide the peg along until the next hole is located by both sound and feel. Then insert the peg vertically into the hole. Continue in a similar manner, taking the pegs from the work area and placing them in the desired holes on the base.

When using bars, the bars may be joined together in the work area or the student's desktop or wherever it seems to be convenient for the student. Then place the interlocked bars in the groove horizontally, with the tip to the left, for a horizontally marked segment or with the tip down for a vertically marked segment. Then slide the interlocked bar in the direction the tip is pointing until it goes into a small hole in the end of the base.

7. Read the numeral on the line segment corresponding to the last peg or bar on the right or top end of the base. (Braille readers should take care to move their fingers perpendicularly in a straight line from the peg or bar to the corresponding numeral on the line segment.)

8. To count the pegs already placed in the base, start at either end and say the corresponding numeral for each one as it is touched while moving along the groove toward the opposite end of the row of pegs. To count the bars already placed in the base, start at either end and say the corresponding numeral for each one until the opposite end is reached. (If difficulty is experienced in perceiving the joining of the bars, they may be pulled apart slightly or all the way for more accurate counting.)

9. Recall or refer back to the problem being worked to see how to proceed to complete it (adding or taking away, comparing to another number, etc.).

10. Using the same scale or another one, perform the necessary operations and/or procedures as outlined above in steps one through seven or in the following suggestions for completing the problem.

SUGGESTIONS FOR USING THE DEVICE FOR TEACHING MATHEMATICS

This device has been designed so that it can be used to augment any mathematics textbook series. Use your own ideas and the following suggestions to get started using the device.

THE NUMBER SYSTEM AND PLACE VALUE

Use segment #1 (0-20 by ones) or an unnumbered segment suitably marked and either pegs or bars to introduce cardinal numbers, counting, and reading numerals.

Use segment #1 (0-20 by ones) with pegs or bars to teach the meaning of zero. There is no hole for a peg corresponding to zero showing that zero has no value. If a bar is placed in the groove with zero on the left, the numeral on the right is correctly read as “one”.

Use segments #1 (0-20 by ones) and #2 (0-200 by tens) with pegs or bars to teach the meaning of ten. Place segment #1 on the bottom of the base and #2 on the

top. Have the student place ten pegs or bars on segment #1, noting that the last corresponding numeral is ten and has two digits. Have the student show the same number ten on segment #2 using only one peg or bar. Notice how this corresponds to the digit "1" in the tens place: 10.

Use segments #1 (0-20 by ones) and #2 (0-200 by tens) with pegs or bars to illustrate expanded notation for two digit numbers and the idea of regrouping. Place segment #1 on the bottom of the base and #2 on the top. Show tens on top and ones on the bottom. Example: 27 = 20 on the top + 7 on the bottom.

Use segment #1 (0-20 by ones) with pegs to show the relationship between odd and even numbers. Place a peg in each hole in the base corresponding to an even number 2, 4, 6, etc. Notice the number of holes skipped (1) between the even numbers and/or the number added (2) to reach the next peg. Follow the same pattern for placing the pegs starting with "1". Note the relationship between the odd and even numbers – odd numbers are those skipped between the even numbers and they follow the same pattern as seen in the first example. The number of holes skipped between the odd numbers is one and the number added to reach the next peg is two. Practice counting by 2's.

Use segment #2 (0-200 by tens) with pegs or bars to become familiar with the relationship among the decades. Segment #1 (0-20 by ones) may also be used to help discover the number which has been added (10)

to get to the next decade on the number line. Practice counting by 10's.

Use segment #3 (0-100 by fives) with pegs or bars to become familiar with the relationship among the fives. Segment #1 (0-20 by ones) may also be used to help discover the number which has been added (5) to get to the next interval on the number line. Practice counting by 5's.

Use segment #4 (-10 – +10 by ones) with pegs or bars to introduce the idea of positive and negative integers. Expand the ideas of "greater than" moving to the right and adding or "less than" moving to the left and subtracting. For example, in the problem 2 – 5 = -3; start on 2, move left 5, and end on -3, showing this to be just an extension of the familiar segments which have zero on the left end. In addition, you may use an unnumbered segment marked like a thermometer and temperatures above and below zero to show the use of positive and negative integers.

SET THEORY

Use segment #1 (0-20 by ones) with either pegs or bars to teach that a set is a collection of objects; to compare sets by one-to-one correspondence of set members emphasizing the corresponding numerals on the segment; and to discover equivalent sets (2 + 3 is another name for 5), empty sets (sets with no members), or subsets (parts of a given set – 2 is part or subset of 5).

Use segment #1 (0-20 by ones) or a segment vertically marked with either pegs or bars to introduce the concepts of "greater than" (moving to the right or up) and "less than" (moving to the left or down) on the number line.

ADDITION AND SUBTRACTION

Use segment #1 (0-20 by ones) with pegs or bars to illustrate and show the relationship of addition and subtraction facts with sums up to 20. A subset of 2 plus a subset of 3 equals a set of 5 or beginning with a set of 5 remove (subtract) a subset of 3 leaving a subset of 2.

Use segment #1 (0-20 by ones) with pegs or bars to demonstrate the commutative (order) principle and the associative (grouping) principle for addition. For example, in the problem 2 + 3 + 1, add a subset of 2 and a subset of 3 on the number line to make a subset of 5 and then add another subset of 1 to make a set of 6 on the number line. Using the same numbers, add a subset of 1 and a subset of 3 on the number line to make a subset of 4 and then add another subset of 2 to make a similar set of 6 on the number line.

Use segments #1 (0-20 by ones) and #2 (0-200 by tens) with pegs or bars to illustrate addition and subtraction of two digit numbers without regrouping and with regrouping. Place segment #1 on the bottom of the base and segment #2 on the top. Show tens on top and ones on the bottom.

Example: 15 + 12 = 27

To show 15, place five pegs on the bottom strip to represent the 5 ones and one peg on the top strip to represent 10. Add 12 by placing two pegs on the bottom strip and one peg on the top strip to represent 10. The number line now shows two pegs on the top strip representing 20 and seven pegs on the bottom strip or 27.

Example: 27 + 14 = 41

To show 27, place seven pegs on the bottom strip and two pegs on the top strip to represent 20. Add 14 by placing four pegs on the bottom strip and one peg on the top strip to represent 10. Regroup the 11 ones on the bottom strip to make one 10 on the top strip for a total of 4 pegs or 40 and leaving one peg on the bottom or 41. Regroup into tens and ones when the sum of the ones is ten or greater. (To help the struggling student understand when to regroup you may place a tactile sticker or graphic art tape after the 9 on segment #1.)

Use segment #2 (0-200 by tens) with pegs or bars to illustrate rounding off numbers. For example, in the problem 17 + 34, round off 17 to 20 (the nearest ten) and place 2 pegs or bars on the first two tens on the number line. Next round off the 34 to 30 (the nearest ten) and add 3 more pegs or bars to the first 2 tens to represent the next 3 tens on the number line. Then finally read the estimated answer for the problem 17 + 34 on the number line: 50.

MULTIPLICATION AND DIVISION

Use segment #1 (0-20 by ones) with pegs or bars to demonstrate that multiplication is repeated addition and that division is repeated subtraction.

Use segment #1 (0-20 by ones) with pegs or bars to illustrate and show the multiplication facts with products up to 20. (If the unnumbered segment with 20 coordinate scale marks is used, this can be extended.) To illustrate the problem 3 x 6 = 18, put six subsets of three together on the number line to make eighteen.

To show the problem 18 ÷ 6 = 3, put a set of 18 pegs, or bars on the number line, remove subsets of 6 until there are less than 6 left and count the number of subsets (3) that were removed.

Use segment #1 (0-20 by ones) with pegs or bars to demonstrate the commutative (order) principle and the associative (grouping) principle for multiplication. For example, in the problem 2 x 3 x 1, put three sets of two pegs on the number line 1 time to make a product of 6 on the number line. Using the same numbers, put one set of 3 on the number line two times to make another similar product of 6 on the number line.

FRACTIONS

Use segments #5 (Halves), #6 (Thirds), #7 (Fourths), and #9 (Sixths) with bars to illustrate the meaning of 1/2, 1/3, 1/4, and 1/6 of a whole. (Use a 1-foot ruler to represent one whole.) Compare the bars to the whole and to each other using the suggested segments on the base two at a time.

Note: Do not use segments #8 (Fifths), #10 (Eighths), or #11 (Fractional Tenths) in this exercise. These segments use a different measure of a "whole".

To compare 1/2 and 1/3, place segment #5 (Halves) on the bottom and segment #6 (Thirds) on the top of the base. Hold the 1-foot ruler next to each of the segments in turn noting that the ruler reaches just to the "1" mark. Put together on the base enough bars (6) to reach the 1/2 mark on the bottom segment. Remove this segment of bars from the base and compare its length to that of the ruler. (It is not the purpose of the ruler to measure, just to represent one whole so reading of the ruler markings is not necessary or even desirable at this time.)

Next, put together on the base enough bars (4) to reach the 1/3 mark on the top segment. Remove this segment of bars from the base and compare its length to that of the ruler. Then compare the bar segment representing 1/2 to the one representing 1/3 away from the base. Finally, place both bar segments back on the base next

to the segments they represent, read, and compare 1/2 and 1/3 on these segments.

Use segments #5 (Halves), #6 (Thirds), #7 (Fourths), and #9 (Sixths) with pegs to illustrate the meaning of 1/2, 1/3, 1/4, and 1/6 of a set having 12 members. Compare the pegs to the set of 12 and to each other using the suggested segments on the base two at a time. For example, to compare 1/2 and 1/3 of the set of 12 place segment #5 (Halves) on the bottom and segment #6 (Thirds) on the top of the base. Find the "1" mark on each segment and count the number of holes in the base needed to reach this mark (12). Put enough pegs (6) in the base to reach the 1/2 mark on the bottom segment. Count the pegs in the base and compare this number to 12. Next, put enough pegs (4) in the base to reach the 1/3 mark on the top segment. Count the pegs in the base and compare this number to 12. Then compare the pegs in 1/2 of a set of 12 (6) to the pegs in 1/3 (4) of a set of 12. Read and compare 1/2 and 1/3 on these segments by sliding the fingers along imaginary perpendiculars between the two segments.

Use segments #5 (Halves), #6 (Thirds), #7 (Fourths), and #9 (Sixths) with bars and pegs to illustrate and compare the meaning of 1/2, 2/4, 3/6, 2/3, 4/6, fractions representing one, equivalent fractions, mixed numbers, and fractions greater than one whole or one set.

Use segments #11 (Fractional Tenths) and #12 (Decimal Tenths) with bars or pegs to introduce the idea of tenths and the use of decimals as an alternate way of

writing tenths. Put segment #11 (Fractional Tenths) on the bottom of the base and #12 (Decimal Tenths) on the top of the base. Find 1/10 on the bottom segment and mark it with a peg or a bar. Following an imaginary perpendicular, slide your fingers to an equivalent place on the top segment and note the marking, ".1". Use the same procedure for 2/10 and .2, 3/10 and .3, etc. Conclude that 1/10 is the same as .1; 2/10 the same as .2; etc.

Use segments #8 (Fifths) and #11 (Fractional Tenths) with bars and pegs to illustrate and compare the meaning of 1/5, 2/10, 3/5, 6/10, 5/5, 10/10, fractions representing one, equivalent fractions, mixed numbers, and fractions greater than one whole or one set.

Use segments #8 (Fifths) and #10 (Eighths) with bars and pegs to illustrate the addition and subtraction of fractions, fractions representing one, mixed numbers, and fractions greater than one whole.

Use segments #7 (Fourths) and #10 (Eighths) to illustrate that fractions will not be equivalent if the whole units are not equivalent. Place segment #7 on the top and segment #10 on the bottom of the base. Have the student find "1" on each segment. While 1/4 should be equivalent to 2/8, demonstrate that 2/8 is actually larger than 1/4 in this example.

MONEY

Use segments #11 (Fractional tenths) and #12 (Decimal tenths) with bars or pegs and dollars, dimes, and pennies to illustrate their meanings and compare the use of the dollar sign and the decimal point with the use of fractions and decimals. Using real money show the student that 10 pennies = 1 dime and 10 dimes = 1 dollar. Compare the value of a penny ($.01) with a dime ($.10) and a dime ($.10) with a dollar ($1.00). Show these amounts (1/10 and .1) on both of the suggested segments using either the pegs or the bars. Note especially the decimal marking on the segment. Write or braille these same amounts using the dollar sign and decimal point. Note the similarities and differences in writing decimals (using the numeric indicator and the decimal point) and writing money (using the dollar sign and the decimal point).

LINEAR MEASUREMENT

Use segment #1 (0-20 by ones) with pegs or bars and a 1-foot ruler to demonstrate measuring by an inch. Since each white plastic cylinder is 1-inch long, the idea of an inch should already be familiar to the student from previous experience with the materials. These white plastic cylinders can be made into various lengths of bars for measuring practice. Place a peg at a particular point dictated by the instructor. Find and point to a similar marking/point on the 1-foot ruler. This will provide practice in reading a ruler.

Use segments #5 (Halves) and #7 (Fourths) with pegs or bars and a 1-foot ruler with half-inch and quarter-inch markings to demonstrate measuring and reading a ruler to the nearest half inch and quarter inch. Use these segments to provide an enlarged version of the markings on the ruler for reading practice as described above.

CLOCKS

Use segments #1 (0-20 by ones) and #3 (0-100 by fives) with pegs or bars to familiarize the student with ones and fives and compare the ones to hours on the clocks and the fives to minutes. The clock is really two circular number line segments – one with visible numbers 1-12 and obvious intervals of 1 hour each, and another where the same numbers 1-12 really stand for the numbers 5-60 and each interval represents 5 minutes.

TEMPERATURE

Use a segment marked vertically to accustom the student to reading a vertical number line. Use segment #4 (-10 – +10) to accustom the student to using negative numbers. Use an unnumbered segment and mark -60° - +130° by tens vertically to resemble a thermometer with bars to represent the mercury adjusted to various temperatures.

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